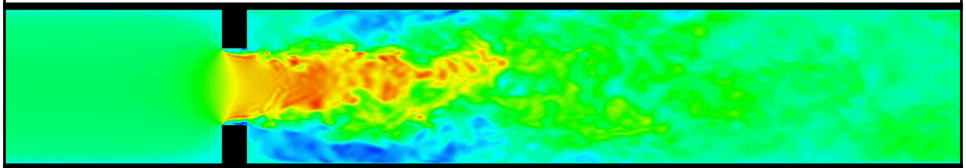
Guided Waves & Flow Interactions



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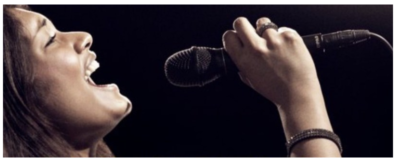
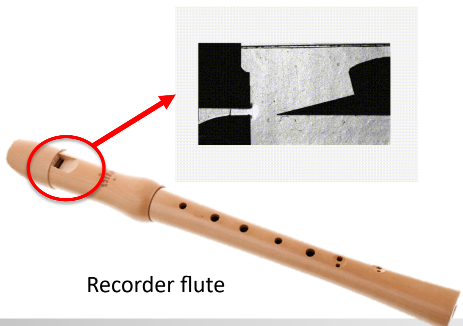
Wave Propagation in Complex and Microstructured Media
Cargèse, Corsica · August 20th - 30th, 2019

Everyone does aeroacoustics (even if you don't know it) …



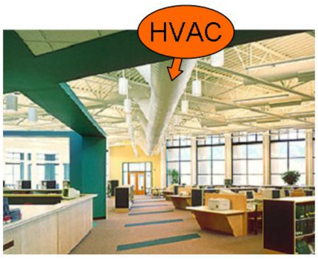
Voice

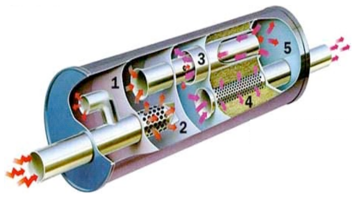


Recorder flute

Wind effect on sound

… and there are many industrial applications





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Outline

Introduction to acoustic with flow

Propagation, convective effects

- Diode
- Black holes

Gain produced by the flow

- Whistling
- Corrugated tubes
- $\mathcal{PT}$-symmetry

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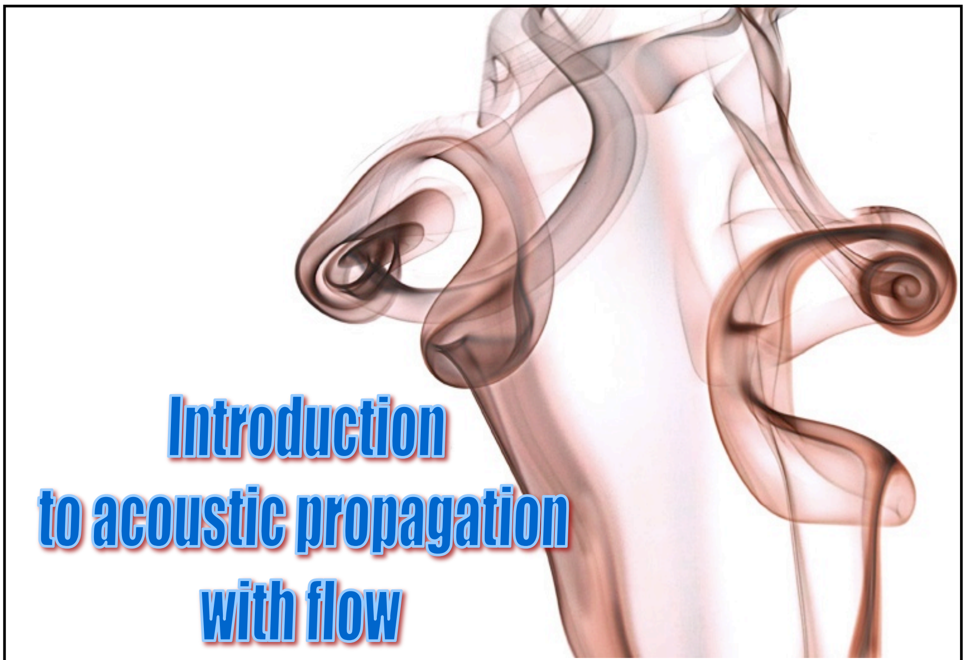
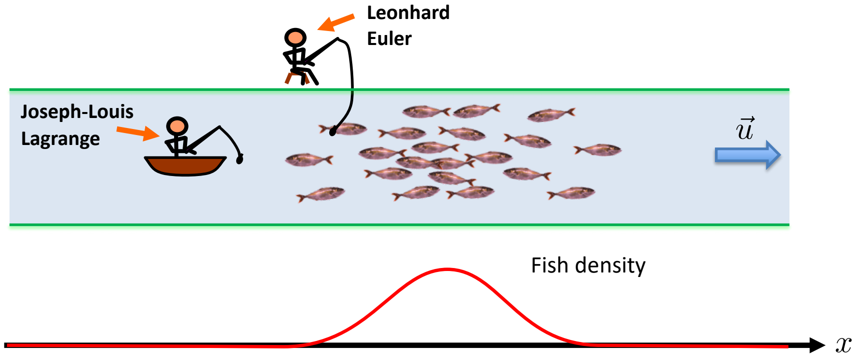


Introduction to acoustic propagation with flow

Convective derivative



Variables are in Eulerian description: $g(\vec{x}, t)$

fixed point

Convective (or total) derivative: $\dfrac{Dg}{Dt} = \dfrac{\partial g}{\partial t} + \vec{u}.\vec{\nabla} g$

General equations

Compressible equations for a perfect fluid

Mass conservation $$\frac{D\rho}{Dt} = -\rho\vec{\nabla}\vec{u}$$

Euler's equation $$\rho\frac{D\vec{u}}{Dt} = -\vec{\nabla}p$$

Isentropic flow $$\frac{D\rho}{Dt} = \frac{1}{c_0^2}\frac{Dp}{Dt}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{u}\,\vec{\nabla}$$

Non linear equations

Linearization without flow

$\vec{u}_0 = \vec{0},\ p_0,\ \rho_0$

$$\vec{u} = \vec{u}_0 + \varepsilon\vec{u}_1$$
$$p = p_0 + \varepsilon p_1$$
$$\rho = \rho_0 + \varepsilon\rho_1$$

Mean values

Perturbations

$\varepsilon \ll 1$

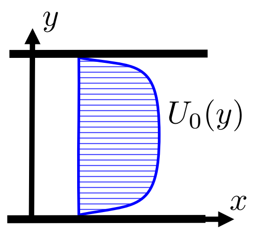
$$\begin{cases} \dfrac{D\rho}{Dt} = -\rho\vec{\nabla}\vec{u} \\ \rho\dfrac{D\vec{u}}{Dt} = -\vec{\nabla}p \\ \dfrac{D\rho}{Dt} = \dfrac{1}{c_0^2}\dfrac{Dp}{Dt} \end{cases}$$

Linearization

$$\begin{cases} \dfrac{\partial\rho_1}{\partial t} = -\rho_0\vec{\nabla}\vec{u}_1 \\ \rho_0\dfrac{\partial\vec{u}_1}{\partial t} = -\vec{\nabla}p_1 \\ p_1 = c_0^2\rho_1 \end{cases}$$

Wave equation $$\frac{1}{c_0^2}\frac{\partial^2 p_1}{\partial t^2} - \triangle p_1 = 0$$

Linearization with incompressible 2D parallel flow



$$\begin{cases} \dfrac{1}{c_0^2}\dfrac{Dp}{Dt} = -\rho \vec{\nabla}.\vec{u} \\ \rho \dfrac{D\vec{u}}{Dt} = -\vec{\nabla} p \end{cases}$$

$$\vec{u} = \underbrace{U_0(y)\vec{x}}_{\vec{u}_0} + \underbrace{u_1(x,y,t)\vec{x} + v_1(x,y,t)\vec{y}}_{\vec{u}_1}$$

Linearization

Order 0 $\begin{cases} \vec{u}_0 = U_0(y)\vec{x} \\ p_0 \text{ and } \rho_0 \text{ constant} \end{cases}$

Linearization with incompressible 2D parallel flow

$$\frac{1}{c_0^2}\frac{Dp}{Dt} = -\rho \vec{\nabla}.\vec{u}$$

Order 1

$$\frac{1}{c_0^2}\left(\frac{\partial p_1}{\partial t} + \vec{u}_0.\vec{\nabla} p_1 + \cancel{\vec{u}_1.\vec{\nabla} p_0}\right) = -\rho_0 \vec{\nabla}\vec{u}_1 - \cancel{\rho_1 \vec{\nabla}\vec{u}_0}$$

$$\frac{1}{\rho_0 c_0^2}\underbrace{\left(\frac{\partial p_1}{\partial t} + U_0 \frac{\partial p_1}{\partial x}\right)}_{\frac{D_0}{Dt} p_1} = -\left(\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y}\right)$$

Linearization with flow in 2D parallel flow

$$\rho \frac{D\vec{u}}{Dt} = -\vec{\nabla} p$$

Order 1

$$\rho_1 \frac{D\vec{u}_0}{Dt} + \rho_0 \frac{D\vec{u}_1}{Dt} = -\vec{\nabla} p_1$$

$$\rho_0 \left(\frac{\partial \vec{u}_1}{\partial t} + \vec{u}_0.\vec{\nabla}\vec{u}_1 + \vec{u}_1.\vec{\nabla}\vec{u}_0 \right) = -\vec{\nabla} p_1$$

$$\frac{D_0}{Dt} u_1 \qquad \rho_0 \left(\frac{\partial u_1}{\partial t} + U_0 \frac{\partial u_1}{\partial x} + v_1 \frac{\mathrm{d}U_0}{\mathrm{d}y} \right) = -\frac{\partial p_1}{\partial x}$$

$$\frac{D_0}{Dt} v_1 \qquad \rho_0 \left(\frac{\partial v_1}{\partial t} + U_0 \frac{\partial v_1}{\partial x} \right) = -\frac{\partial p_1}{\partial y}$$

Linearization with flow in 2D parallel flow

$$\begin{cases} \rho_0 \dfrac{D_0}{Dt} u_1 + \rho_0 \dfrac{\mathrm{d}U_0}{\mathrm{d}y} v_1 = -\dfrac{\partial p_1}{\partial x} \\ \rho_0 \dfrac{D_0}{Dt} v_1 = -\dfrac{\partial p_1}{\partial y} \\ \dfrac{1}{\rho_0 c_0^2} \dfrac{D_0}{Dt} p_1 = -\left(\dfrac{\partial u_1}{\partial x} + \dfrac{\partial v_1}{\partial y} \right) \end{cases}$$

$$\begin{cases} \dfrac{1}{c_0^2} \dfrac{D_0^2}{Dt^2} p_1 - \nabla^2 p_1 = 2\rho_0 \dfrac{\mathrm{d}U_0}{\mathrm{d}y} \dfrac{\partial v_1}{\partial x} \\ \rho_0 \dfrac{D_0}{Dt} v_1 = -\dfrac{\partial p_1}{\partial y} \end{cases}$$

Linked to flow vorticity

$$\vec{\omega}_0 = \vec{\nabla} \times \vec{u}_0 = \frac{\mathrm{d}U_0}{\mathrm{d}y} \vec{z}$$

For shear flow, there is no longer a propagation equation

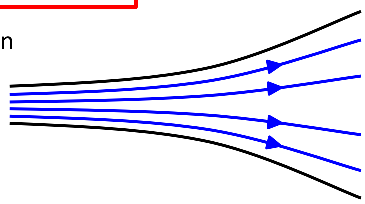
Linearization in an irrotational flow

Irrotational mean flow

$\vec{\omega}_0 = \vec{\nabla} \times \vec{u}_0 = \vec{0}$ → Irrotational perturbations (if the initial rotation is 0) $\vec{u}_1 = \vec{\nabla}\psi$

$$\frac{D_0}{Dt}\left(\frac{1}{c_0^2}\frac{D_0}{Dt}\psi\right) - \frac{1}{\rho_0}\vec{\nabla}\left(\rho_0 \vec{\nabla}\psi\right) = 0$$

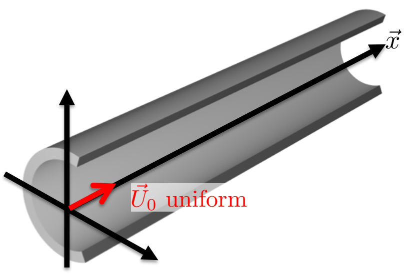
Distorted wave equation



Convective effects:
Breaking of time reversal symmetry

The simplest example of an irrotational flow:

Uniform flow
in a straight duct



$$\frac{1}{c_0^2}\frac{D_0^2 p}{Dt^2} - \Delta p = 0$$

$$p = \hat{p}\, e^{(-\mathrm{j}\omega t + \mathrm{j}kx)}$$

$$\begin{cases} \Delta_\perp \hat{p} + \alpha^2 \hat{p} = 0 \\ + \text{ BC: } \vec{\nabla}\hat{p}.\vec{n} = 0 \text{ at the wall} \end{cases}$$

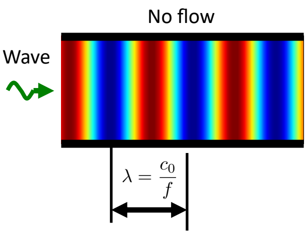
$$\alpha^2 = \left(\frac{\omega}{c_0} - kM\right)^2 - k^2$$

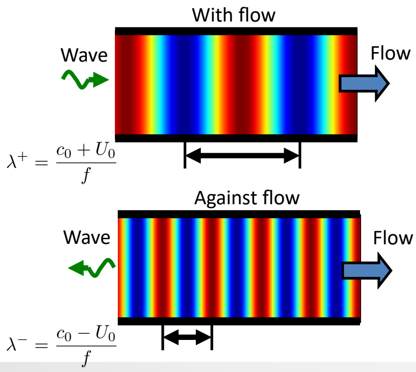
$\hat{p}$ uniform with $\alpha = 0$ is always solution

The simplest example of an irrotational flow

Plane wave solution:

$$\alpha^2 = \left(\frac{\omega}{c_0} - kM\right)^2 - k^2 = 0 \qquad \begin{cases} k^+ = \dfrac{\omega}{c_0(1+M)} \\ k^- = -\dfrac{\omega}{c_0(1-M)} \end{cases}$$





The simplest example of an irrotational flow

Plane wave solution:

$$p(x) = p^+ \, e^{\mathrm{j}k^+ x} + p^- \, e^{-\mathrm{j}k^- x}$$

$$k^+ = \frac{k_0}{1+M}$$

$$k^- = \frac{k_0}{1-M}$$

Using Euler Eq. $\rho_0 \dfrac{Du}{Dt} = -\dfrac{\partial p}{\partial x}$

$$u(x) = \frac{1}{\rho_0 c_0}\left(p^+ \, e^{\mathrm{j}k^+ x} - p^- \, e^{-\mathrm{j}k^- x}\right)$$

Linearization in a rotational flow

Fluctuating velocity $\vec{u}_1$

Fluctuating displacement $\vec{\delta}_1$

Mean flow vorticity $\vec{\omega}_0$

Irrotational part

Rotational part due to flow vorticity

$$\vec{u}_1 = \vec{\nabla}\psi + \vec{\delta}_1 \times \vec{\omega}_0$$

$$\frac{D_0}{Dt}\left(\frac{1}{c_0^2}\frac{D_0}{Dt}\psi\right) - \frac{1}{\rho_0}\vec{\nabla}\left(\rho_0\vec{\nabla}\psi\right) = \frac{1}{\rho_0}\vec{\nabla}\left(\rho_0\vec{\xi}\right)$$

$$\frac{D_0}{Dt}\vec{\xi} = \vec{\nabla}\psi \times \vec{\omega}_0 - (\vec{\xi}\vec{\nabla})\vec{u_0}$$

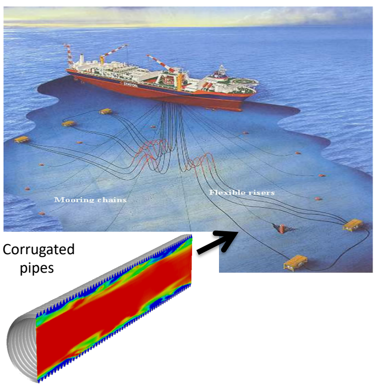
Goldstein equations

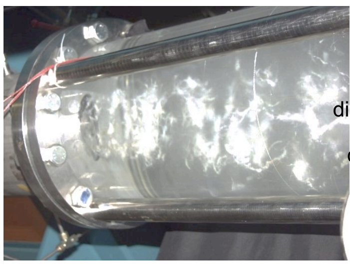
With $\vec{\xi} = \vec{\delta}_1 \times \vec{\omega}_0$

Could lead to strong coupling
Acoustic ↔ Vorticity

Acoustic in rotational flows

Could lead to strong coupling
Acoustic ↔ Vorticity

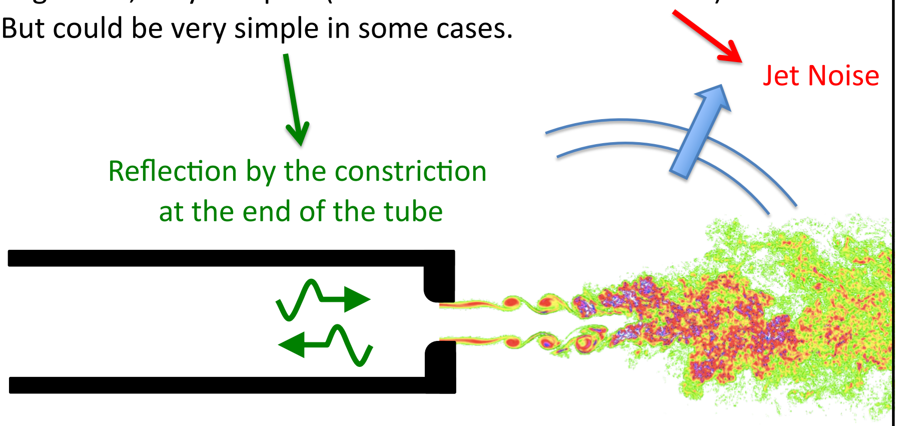




Whistling in a diaphragm in water visualized by cavitation (EDF)

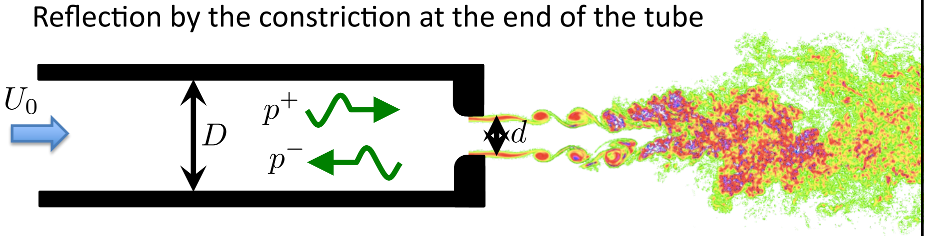
Acoustic in rotational flows

In general, very complex (need of numerical simulation)
But could be very simple in some cases.



Acoustic in rotational flows

Reflection by the constriction at the end of the tube



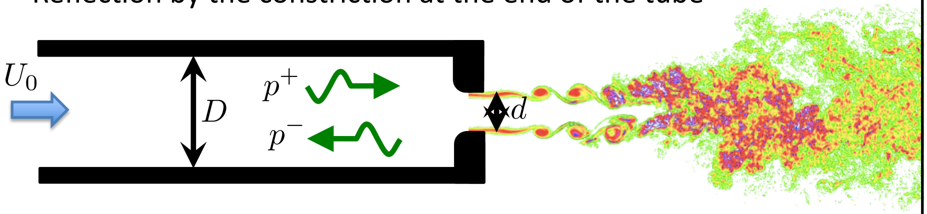
Mass conservation $D^2 u_t = d^2 u_j$

Bernoulli equation $p_t + \rho_0 \frac{u_t^2}{2} = p_j + \rho_0 \frac{u_j^2}{2} = P_{atm} + \rho_0 \frac{D^4}{d^4} \frac{u_t^2}{2}$

$$p_t - P_{atm} = \rho_0 \left(\frac{D^4}{d^4} - 1 \right) \frac{u_t^2}{2}$$

Acoustic in rotational flows

Reflection by the constriction at the end of the tube



$$p_t - P_{atm} = \rho_0 \left(\frac{D^4}{d^4} - 1 \right) \frac{u_t^2}{2} \quad \xrightarrow{\text{Linearization}} \quad p'_t = \rho_0 \left(\frac{D^4}{d^4} - 1 \right) U_0 u'_t$$

$$p^+ + p^- = \rho_0 \left(\frac{D^4}{d^4} - 1 \right) U_0 \frac{p^+ - p^-}{\rho_0 c_0}$$

$$r_E = \frac{p^-}{p^+} = -\frac{1 - (D^4/d^4 - 1)M}{1 + (D^4/d^4 - 1)M}$$

Can be used to make an anechoic termination

Acoustic in rotational flows

Reflection by the constriction at the end of the tube