LINEAR STABILITY OF MEAN FLOWS AND FREQUENCY PREDICTION

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The frequency of the von Kármán vortex street can be predicted by linear stability analysis around its mean flow. Barkley [1] has shown this to yield an eigenvalue whose real part is zero and whose imaginary part matches the nonlinear frequency This property was named RZIF by Turton et al. [2]; moreover they found that the traveling waves (TW) of thermosolutal convection have the RZIF property as shown in Figure (1). They explained this as a consequence of the fact that the temporal Fourier spectrum consists primarily of the mean flow and first harmonic. From this same idea Mantič-Lugo et al. [3] developed the Self-Consistent Model (SCM)

$$0 = \mathcal{L}\bar{U} + \mathcal{N}(\bar{U},\bar{U}) + \mathcal{N}(u_1,u_1^*) \tag{1a}$$
$$(\sigma + i\omega)u_1 = \mathcal{L}_{\bar{U}}u_1 \tag{1b}$$
$$\|u_1\| = A, \qquad \sigma = 0 \tag{1c}$$

for the base flow $\bar{U}$, the complex eigenvector u_1 and eigenvalue $\sigma + i\omega$, and its amplitude A. Mantič-Lugo et al. [3] solved these equations iteratively for the cylinder wake for each value of A by determining $\bar{U}$ via Newton's method from (1a), then determining u_1 and $\sigma + i\omega$ via diagonalization of (1b), and finally choosing the value of A such that $\sigma = 0$. We have carried out the same calculation for the traveling waves of thermosolutal convection, but we were able to obtain convergence of this procedure only up to $r = 2.25$. We then implemented a full Newton's method to solve the coupled problem (1) up to at least $r = 3$. Figure 1 shows that while the RZIF property is satisfied up to at least $r = 3$, the SCM model reproduces the exact frequency only for $r < 2.1$ and deviates entirely from it for $r > 2.5$. Thus, the nonlinear interaction of u_1 with itself yields a mean flow which is insufficiently accurate. Our next step will be to take into account higher harmonics and to apply this analysis to the standing waves, for which RZIF does not hold.

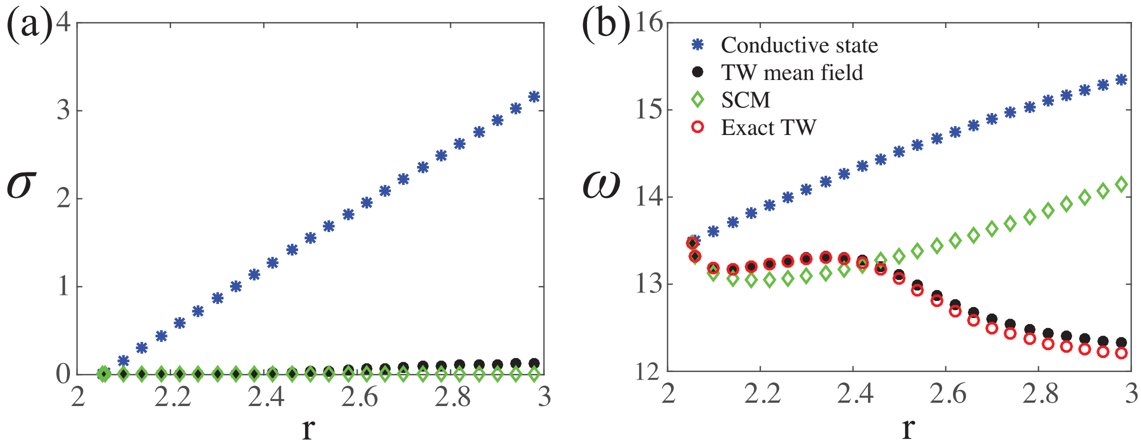

Figure 1. Growth rate (a) and frequency (b) as a function of Rayleigh number r for TW branch of thermosolutal convection. Linearization about the mean field yields eigenvalues (black filled circles) whose real part is close to zero and whose imaginary part is close to the exact nonlinear frequency (hollow red circles), i.e. the RZIF property is satisfied over the range shown. The Self-Consistent Model yields eigenvalues (green diamonds) whose real part is zero by construction, but whose imaginary part is close to the exact nonlinear frequency only for small r. The eigenvalues obtained by linearization about the conductive state are shown as blue stars.

References

[1] D. Barkley, Linear analysis of the cylinder wake mean flow. *Europhys. Lett.*, **75**: 750–756, 2006.

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[3] V. Mantič-Lugo, A. Cristóbal, and F. Gallaire, Self-consistent mean flow description of the nonlinear saturation of the vortex shedding in the cylinder wake. *Phys Rev Lett*, **113**: 084501, 2014.