Coupled-resonator phononic waveguides
Coupled-resonator acousto-elastic waveguides – 1

Figure: The blue, gray and white parts represent water, aluminum and vacuum, respectively [46].
Coupled-resonator acousto-elastic waveguides

Figure: Displacement and pressure fields of the six defect modes of the acousto-elastic phononic crystal, shown at the $\Gamma$ point of the first Brillouin zone.
The dispersion relation of CRAEW modes is very smooth. This property can actually be associated with the rapid decrease with distance of the coupling strength between adjacent cavities [47]. The dispersion can be expressed directly as the Fourier series

$$\omega(k) = \Gamma_0 + \sum_{m=1}^{\infty} 2\Gamma_m \cos(km\Lambda)$$

(1)

The Fourier coefficients $\Gamma_m$ can be interpreted as representing the coupling strength between defects separated by a distance $m\Lambda$.

Note that periodicity of the waveguide alone implies the Fourier series expansion: the expression is valid for all phononic crystal waveguides.
Coupled-resonator acousto-elastic waveguides

Figure: 8-bend waveguide: there are no significant losses at a waveguide bend within each passband.
Figure: Splitter circuit: the symmetry of the Bloch wave allows for even splitting.
Channeled spectrum – 1

Figure: Different coupled-resonator waveguides to test the channeled spectrum idea. Square-lattice sonic crystal of mercury cylinders in water. (a) $L = 10\Lambda$, (b) $L = 13\Lambda$, (c) $L = 17\Lambda$, and (d) $L = 13\Lambda$ [48].
Coupled-resonator phononic waveguides

Channeled spectrum

Figure: Channeled spectrum: sequence of frequency maxima and minima in the transmission spectrum. It depends mostly on the waveguide length.
Figure: (a) Simplified model of transmission through a single-mode periodic waveguide at a single frequency. (b) Graphical construction of the channeled spectrum from the dispersion relation of the infinite waveguide.
Superposition of a left-traveling Bloch wave, $p_l(x,y)$, with a right-traveling Bloch wave, $p_r(x,y)$,

$$p(\omega; x, y) = \alpha p_r(x, y)e^{-ik(\omega)x} + \beta p_l(x, y)e^{+ik(\omega)x}$$  \hspace{1cm} (2)

gives transmission:

$$t(\omega) = \alpha e^{-i(k(\omega)L} + \beta e^{+i(k(\omega)L}.$$  \hspace{1cm} (3)

The transmission in intensity is then

$$|t(\omega)|^2 = |\alpha|^2 + |\beta|^2 + 2|\alpha\beta|\cos(2k(\omega)L - \theta),$$  \hspace{1cm} (4)

with $\theta = \text{Arg}(\alpha\beta^*)$ a phase angle. Transmission maxima are obtained when $2k(\omega)L = \theta$ modulo $2\pi$, or

$$k(\omega_n)\Lambda = \frac{\theta}{2N} + \frac{n}{N}\pi.$$  \hspace{1cm} (5)
Figure: Pressure distribution of the straight CRAW (CW1, $L = 10\Lambda$) at the resonance peaks. The number of pressure oscillations is shown below the field maps.
Coupled-resonator circuits – 1, stainless steel

Figure: Phononic crystal slab of cross holes in stainless steel. The parameters $b/a=0.9$, $c/a=0.2$ and $h/a=0.4$, with the lattice constant $a=20$ mm optimize the band gap width [49].
Coupled-resonator circuits – 2, perfect crystal
**Guided waves in phononic crystals**
- Coupled-resonator phononic waveguides
- Coupled-resonator circuits

**Coupled-resonator circuits – 3**

![Graph showing transmission and frequency response](image)

- **Graph (a)**
  - Frequency (kHz) range: 50 to 100
  - Transmission (dB) range: -15 to 15

- **Graph (b)**
  - Frequency: 62.19 kHz
  - Transmission (dB) range: -15 to 15
  - Maxima: 60nm, 50nm

- **Graph (c)**
  - Frequency: 73.44 kHz
  - Transmission (dB) range: -15 to 15
  - Maxima: 10nm, 25nm, 40nm, 15nm

- **Graph (d)**
  - Frequency: 87.81 kHz
  - Transmission (dB) range: -15 to 15
  - Maxima: 40nm, 15nm