Transformation optics and its applications to antennas

Outline

• Theory:
  • Transformation optics concept.
  • Types:
    • Analytical transformation.
    • Quasi-conformal transformation.
    • Non-Euclidean transformation.
• Practice:
  • Lens design.
    • Compressed lenses.
    • Planar Lenses.
    • Collimated lenses.
    • Bespoke lenses.
  • Surface propagation.
    • Cloaking.
    • Surface waves lensing.
• Conclusions.
Transformation Optics Concept

- In 2006 two pioneering papers were published in *Science* defining the concept of **transformation optics**:
  - According to the theory, any given electromagnetic device can be transformed into an infinite number of new ones with same electromagnetic responses.
  - This tool has incredible possibilities to redesign classic devices.

\[
\varepsilon' = \frac{J \varepsilon J^T}{|J|}
\]

\[
\mu' = \frac{J \mu J^T}{|J|}
\]

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- **Conclusions.**
1) Analytical Transformation

• To make an analytical transformation taking into account all the components in Maxwell’s equations.
• No approximations:
  - The physical space has the same response as the original virtual space.
• In practice, this idea is unaffordable:
  - Dispersive materials.
  - Anisotropic materials.

1) Analytical Transformation:
Dispersive materials and anisotropy

• Lets assume the simplest transformation:
  - Compression in one of coordinate axes.

\[
x'(x) = \begin{cases}
  ax + c, & l_1 < x < l_2 \\
  l_1' + (x - l_1), & x \leq l_1 \\
  l_2' + (x - l_2), & x \geq l_2
\end{cases}
\]

• The new permittivity and permeability maps will be:

\[
\begin{bmatrix}
  \varepsilon' &=& \begin{bmatrix}
  a & 0 & 0 \\
  0 & 1/a & 0 \\
  0 & 0 & 1/a
  \end{bmatrix}
  \\
  \mu' &=& \begin{bmatrix}
  a & 0 & 0 \\
  0 & 1/a & 0 \\
  0 & 0 & 1/a
  \end{bmatrix}
\end{bmatrix}
\]

Dispersive materials

- Materials with refractive indexes lower than 1 can be only obtained with the use of metamaterials.
- Metamaterials are strongly dispersive:
  - Very limited bandwidth of operation.

\[ \varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - j\omega\gamma} \]

- To avoid the use of these materials is always an advantage for practical applications.

2) Discrete Transformation

- To make a transformation based on graphical coordinates.
- Importance of the coordinate lines to be orthogonal to the metallic boundaries.
- Transformation based on areas (not in shapes).
  - It does not take into consideration non-linear effects.

\[ n' = n_0 \left( \frac{\Delta x_0 \Delta y_0}{\Delta x_0 \Delta y_0} \right) \]

2) Discrete Transformation: Dispersive materials (I)

- Depending on the geometry, the required refractive index for the new map will have some lower than one index regions.
  - Lower than 1 refractive indexes require metamaterials implementations.
  - Dispersive materials and narrow band.

2) Discrete Transformation: Dispersive materials (II)

- Two possible solutions:
  1. To develop the transformation over an original dense material.
  2. To approximate these values to 1.
### 3) Non-Euclidean transformation

- To analyse the ray paths in a surface and to obtain the equivalent 2D plane which remains the same properties.

- One method: Stereographic projection

- Maxwell Fish Eye

- Luneburg

- Eaton

- 90° rotation


\[ \text{dS}_1 = n \cdot \text{dS}_2 \]

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### Maxwell Fish Eye Lens

- It is a lens in which a source in any excited point of the circle surface will converge exactly at the opposite size of the circle.

- It is a rotationally symmetric lens.

- Equivalent to an homogeneous sphere.

\[ n(\rho) = \frac{2n_0}{1 + \left( \frac{\rho}{a} \right)^2} \]

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Compressed lenses: Transformation optics

- Using transformation optics, we can compress the space:

\[
\varepsilon' = \varepsilon \\
\mu' = \mu \\
\varepsilon = \varepsilon_0 \\
\mu = \mu_0 \left( 2 - \frac{(\alpha x^2 + \gamma y^2)}{R^2} \right)
\]

Compressed Luneburg lens

• Main direction:


Compressed Luneburg lens

• Oblique radiation pattern:

Mode-matching: Anisotropy

- Lens compression:

\[ n_{\text{Glide}}^{\text{MAX}} = 2.86 \Rightarrow \beta_{\text{Glide}}^{\text{MAX}} = 43\% \]

\[ n_{\text{Non-Glide}}^{\text{MAX}} = 2.05 \Rightarrow \beta_{\text{Non-Glide}}^{\text{MAX}} = 3.5\% \]


Planar lenses

- Option 1:
  - Fresnel Lenses
  - Single frequency of operation


- Option 2:
  - Transformation Electromagnetics
  - UWB solution.
Design Process

- Quasi-conformal Transformation Optics.
- A discretization for the manufacturing process is required.

Discretization process

- Spheres/Ellipsoids:
  - Ellipsoidal/spherical discretization along the iso-permittivity lines.
  - Zones of different permittivities:
    - $2 < \varepsilon_r < 14.5$
    - 10-15 zones
Comparison with Fresnel lens

- Our solution overcomes Fresnel lenses in Bandwidth.
  - That is the other existing planar version.

Manufacturing

- Alternative permittivity regions have been produced through a combination of tailoring:
  1. Particle size.
  2. Dispersion and volume fraction of materials.
- The particle sizes where obtained using particle size reduction methods such as milling to achieve the distribution of sizes required, and these varied from nano- to micron size.
Stability with Frequency

- The materials have a constant response with frequency (from 8GHz to 13GHz).

Measurements

- Low frequency (1-7GHz):
  - Anechoic chamber.

- High frequency (7-14GHz):
  - Near Field Scanner
  - NSI-200V-3x3
Simulation vs Measurements

- Good agreement in terms of phase and amplitude (10 GHz)

![Simulation and Measurement](image)


Collimated lenses

- Quasi-conformal transformation to obtain collimated lenses: multi-beams.

From cylindrical to squared waves

- Optical transformations can be used to create completely new type of lenses.
  - Transformation of a cylindrical wave in four directive beams.

Bespoke lenses

- To produce ad-hoc lens for practical feedings.
- Most common situation is to produce a plane wave from a given radiator:
  - To increase the directivity.
Bespoke lenses: Aperture antenna (I)

• Technique: Obtaining the map for the lens.


Bespoke lenses: Aperture Antenna (II)

• Technique: Final design and results.

Bespoke lenses: Spiral antenna

- The lens is not limited in bandwidth and it applies to both polarizations:


Bespoke lenses: Slot lines

- The lens is not limited in bandwidth.


Bespoke lenses: Slot lines

- Figures of merit.

![Directivity Graph]

- **Directivity**
  - Designed bespoke lens
  - Hyper-hemispherical lens without matching layers
  - Max. directivity – aperture of the designed bespoke lens

![Cross-polarization Graph]

- **Cross-polarization**
  - Designed bespoke lens
  - Hyper-hemispherical lens with matching layers
  - Hyper-hemispherical lens without matching layers

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- **Conclusions.**
**Theory**

- The refractive index distribution of a rotationally symmetric curved surface can mimic an equivalent a flat homogeneous surface:

\[
\int_0^{\frac{\pi}{2}} n(\theta) \sqrt{R(\theta)^2 + R'(\theta)^2} \, d\theta = \int_0^r \, dr
\]

\[
2\pi R(\theta) \sin(\theta) n(\theta) = 2\pi r
\]

\[
n'(\theta) = \frac{\sqrt{R(\theta)^2 + R'(\theta)^2} - R'(\theta) \sin \theta - R(\theta) \cos \theta}{R(\theta) \sin \theta}
\]

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**Results**

- Cloaking in a thin metallic cavity:

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Surface waves implementation

- Slabs of different dielectric constants with constant thickness.

Surface waves results

- Very robust design.
- Only 7 layers have been used.
- Small deviations in the manufacturing process would not seriously influence the performance.
Experimental results

- Implementation with a single dielectric slab which changes the with the position to achieve different equivalent refractive indexes.


Non-Euclidian mapping

- By equating optical path lengths, it is possible to calculate the refractive index of a lens on a curved surface.

\[
\begin{align*}
\text{Flat space} & \\
\mathbf{R} & = \begin{pmatrix} n_1(r) \\ s_1 \end{pmatrix} \\
\mathbf{R} & = \begin{pmatrix} n_2(\theta) \\ s_2 \end{pmatrix} \\
\mathbf{R}' & = \begin{pmatrix} R' \sin(\theta) \\ R' \cos(\theta) \end{pmatrix} \\
\int_{0}^{\pi/2} n_2(\theta) \sqrt{R(\theta)^2 + R'(\theta)^2} d\theta & = \int_{0}^{\pi/2} n_1(r) dr \\
2\pi \cdot \mathbf{R} \cdot n_1(\theta) \sin(\theta) & = 2\pi \cdot \mathbf{R} \cdot n_1(r) \\
n_2(\theta) & = \left( 1 + \frac{n_1'(r)}{n_1(r)} \right) \sqrt{R(\theta)^2 + R'(\theta)^2} - R'(\theta) \sin(\theta) - R(\theta) \cos(\theta) \\
n_2(\theta) & = \frac{R'(\theta) \sin(\theta)}{R(\theta) \sin(\theta)}
\end{align*}
\]
**Luneburg lens examples**


$n(r) = \sqrt{2 - \left(\frac{2}{a}\right)^2}$

$n(\theta) = \frac{\sqrt{1 + 3\cos(\theta)}}{(1 + \cos(\theta))^{\frac{1}{2}}}$


---

**Maxwell fish eye lens examples**


$n(r) = \frac{n_0}{1 + r^2}$

Lens implementation

• A Luneburg lens can be implemented with this technique.
• The surface can be bended to reduce the vertical dimension.


Antenna design

• Waveguides as connectors.
• Optimized flare design.

Luneburg lens antenna

- The antenna is fed with coaxial ports.
- Each port excitation produces a different beam.


Matching

- Slightly higher mismatch in measurements

Radiation patterns

- Good agreement with simulations

**Simulations**

**Measurements**


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Conclusions

- The concept of Transformation Optics has been introduced.
- Three possible methodologies: Euclidean (analytical and discrete) and non-Euclidean have been drawn, and their advantages and disadvantages have been summarized.
- Few examples of design have been introduced:
  1. Conformal lenses:
     - They can be used to design lenses which bespoke surfaces.
  2. Planar lenses:
     - The use of metamaterials is not necessary for this design.
     - Measurements corroborate the original results.
  3. Surface propagation:
     - Cloaking has been demonstrating to be obtain with only full dielectric materials.
     - This technique can be employed to produce lenses conformal to surfaces and to eliminate singularities of lenses.

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